* H M - 0 0 3 - 0 1 6 3 0 3 *

HM-003-016303

Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

May / June – 2017

Mathematics : Paper - 3003

(Number Theory - 1) (New Course)

Faculty Code : 003

Subject Code : 016303

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions in this paper.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries 2 Marks)

(i) If p is a prime number and n is a positive integer then the number of positive integers relatively prime to p^n is ________.

(ii) If p and q are distinct primes then g.c.d. of p^m and q^n is ________. $(m,n \in \mathbb{N})$

(iii) If $p = 79$ then $x^2 + 1 \equiv 0 (\text{mod } p)$ has ________ solutions.

(iv) If p is a prime number and n is a positive integer then the number of positive divisors of p^n =________.

(v) If $n = 100 \times 99$ then $\varnothing(n)$ = ________.

(vi) If p is a prime number and p does not divide a then $a^p \equiv$ ________ $(mod p)$.

(vii) If m divides ab then $\frac{m}{(a,\, m)}$ divides ________.

2 Attempt any **two** :

(i) Prove that there are infinitely many prime integers. **7**

(ii) Write the statement of division algorithm and prove it. **7**

(iii) Suppose $(a,m) = 1$ then prove that $a^{\phi(m)} \equiv 1 (\text{mod}\, m)$ **7**

3 Attempt all the questions :

(i) Write the statement of Chinese remainder theorem and prove it. **6**

(ii) Find the smallest positive integer x such that the remainder is 3 when it is divided by 5, the remainder is 5 when it is divided by 7 and the remainder is 7 when it is divided by 11. **4**

(iii) If $n \neq 0, a, x, y$ are integers then prove that **4**

$ax \equiv ay \pmod{m}$ if and only if $x \equiv y\left(\bmod \frac{n}{(a,n)}\right)$

OR

3 Attempt all the questions :

(i) Suppose $f(x)$ is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 \pmod{p}$ has degree n. Prove that $f(x) \equiv 0 \pmod{p}$ has at most n solutions in any complete residue system $(\bmod\, p)$. **7**

(ii) First find the solutions of $f(x) \equiv 0 \pmod{2}$, $f(x) \equiv 0 \pmod{5}$, $f(x) \equiv 0 \pmod{7}$ and use them to find all solutions of $f(x) \equiv 0 \pmod{70}$. Here $f(x) = x^2 - 1$. **7**

4 Attempt any **two** :

(i) Determine which of the following have primitive roots and if an integer has a primitive root then find at least two primitive roots : 5, 25, 106 and 35. **7**

(ii) Suppose n is an odd positive integer and a is an odd integer. Prove that $x^n \equiv a \left(\bmod 2^j\right); (j > 3)$ has a unique solution. **7**

(iii) Prove that $\sum_{d/n} \varnothing(d) = n$, for any positive integer n. **7**

5 Do as directed : (Each carries 2 marks)

(i) Write the definition of Mobius function.

(ii) Find the value of $\varnothing(25)$ using Mobius inversion formula.

(iii) Find the highest power of 79 which divides 47321!

(iv) Find the number of positive divisors of 180.

(v) Find the values of $\omega(n)$ for $n = 23, 77, 101, 109$.

(vi) Give an example of a multiplicative function which is not totally multiplicative.

(vii) Find $\varnothing(n)$ for $n = 250, 303$ and 196.